Chapter 4

Congruent Triangles

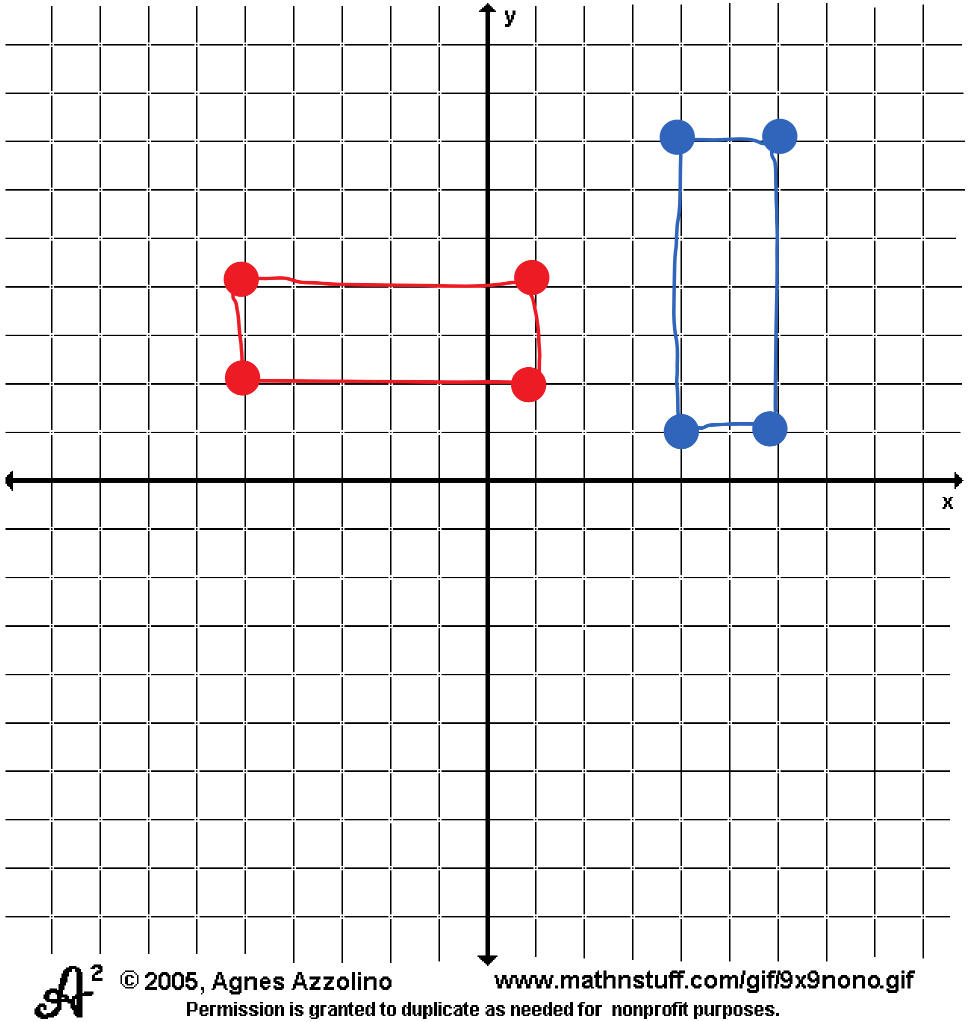
Section 7
Triangles and Coordinate Proof

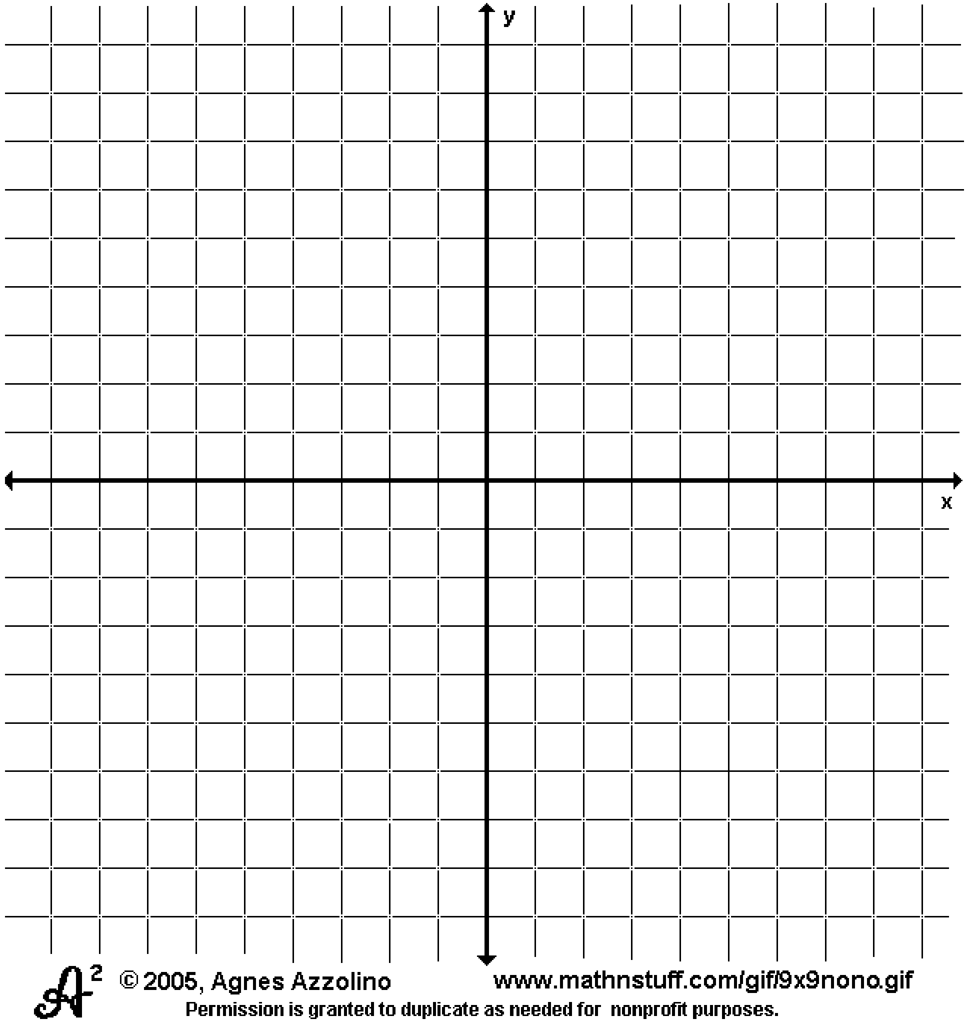
GOAL 1: Placing Figures in a Coordinate Plane

So far, you have studied two-column proofs, paragraph proofs, and flow proofs. A **coordinate proof** involves placing geometric figures in a coordinate plane. Then you can use the Distance Formula and the Midpoint Formula, as well as postulates and theorems, to prove statements about the figures.

Example 1: Placing a Rectangle in a Coordinate Plane

Place a 2-unit by 6-unit rectangle in a coordinate plane.

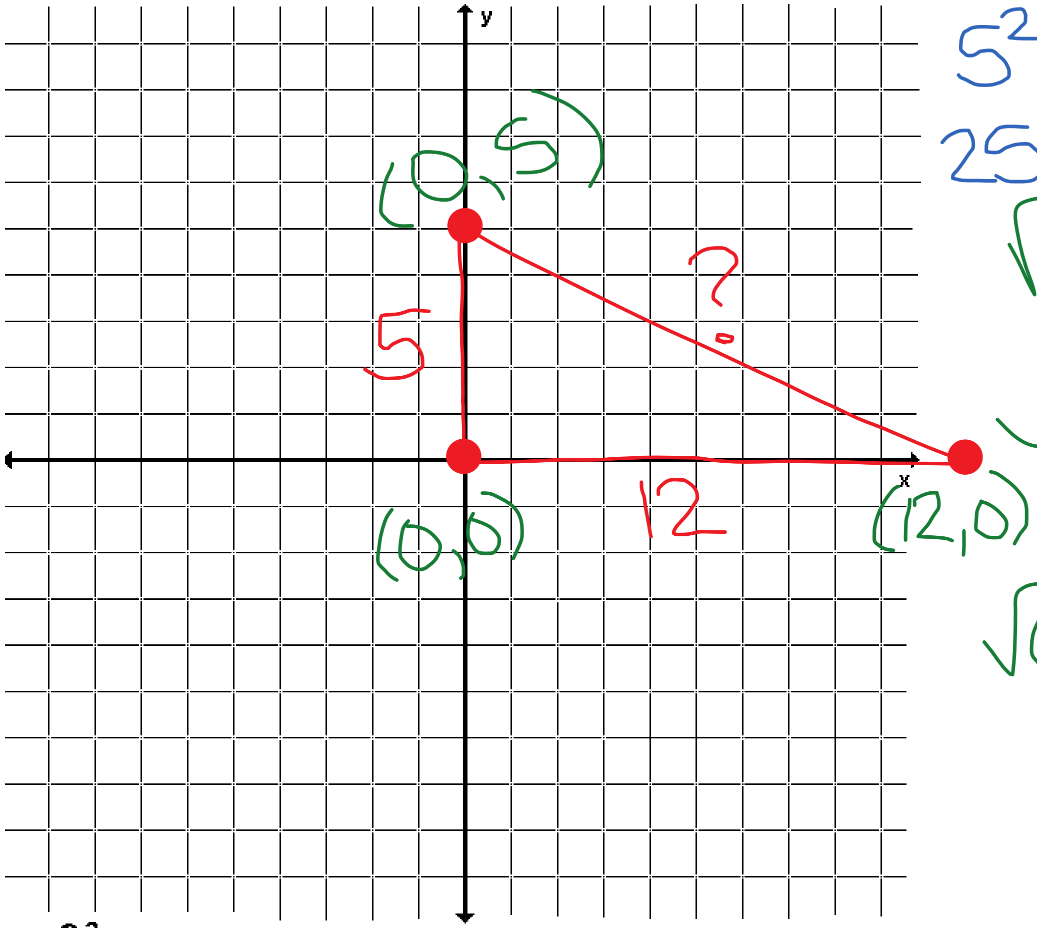




Once a figure has been placed in a coordinate plane, you can use the Distance Formula or the Midpoint Formula to measure distances or locate points.

Example 2: Using the Distance Formula

A right triangle has legs of 5 units and 12 units. Place the triangle in a coordinate plane. Label the coordinates of the vertices and find the length of the hypotenuse.



$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

$\sqrt{169} = \sqrt{c^2}$

$13 = c$

$\sqrt{(12-0)^2 + (0-5)^2}$

$\sqrt{12^2 + (-5)^2}$

$\sqrt{144 + 25}$

$\sqrt{169} \Rightarrow 13$

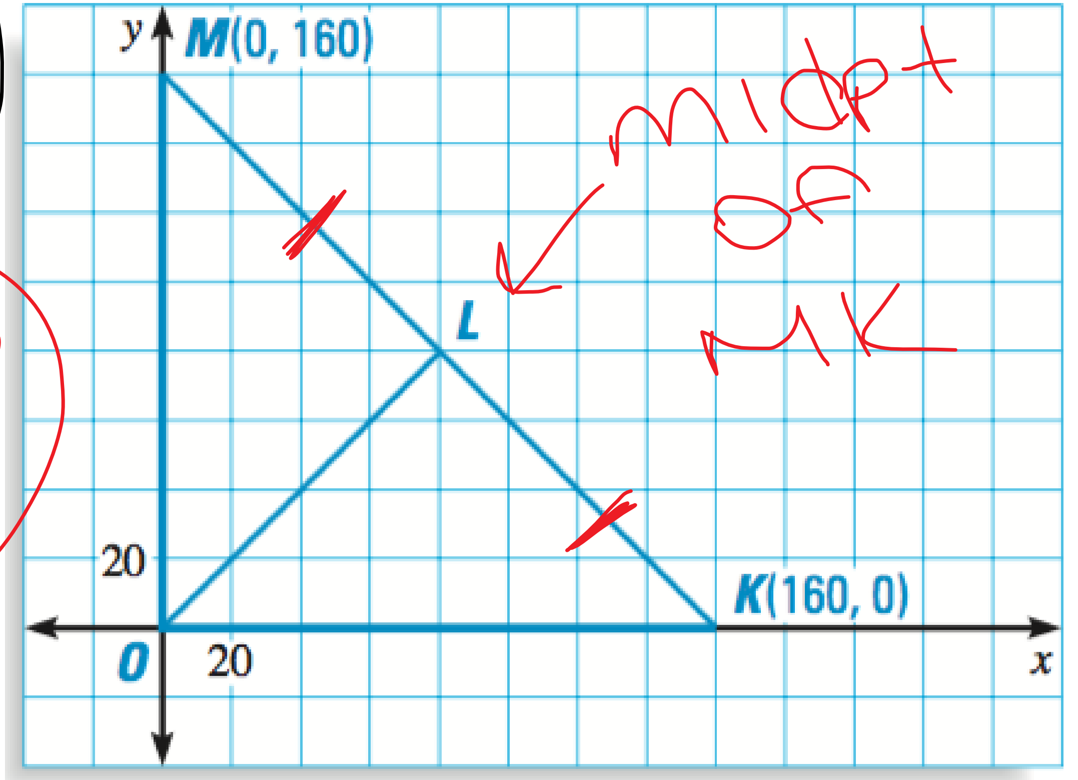
Example 3: Using the Midpoint Formula

In the diagram, $\Delta MLO \cong \Delta KLO$. Find the coordinates of point L.

$$\left(\frac{x+x}{2}, \frac{y+y}{2} \right)$$

$$\left(\frac{0+160}{2}, \frac{160+0}{2} \right)$$

$$(80, 80)$$



GOAL 2: Writing Coordinate Proofs

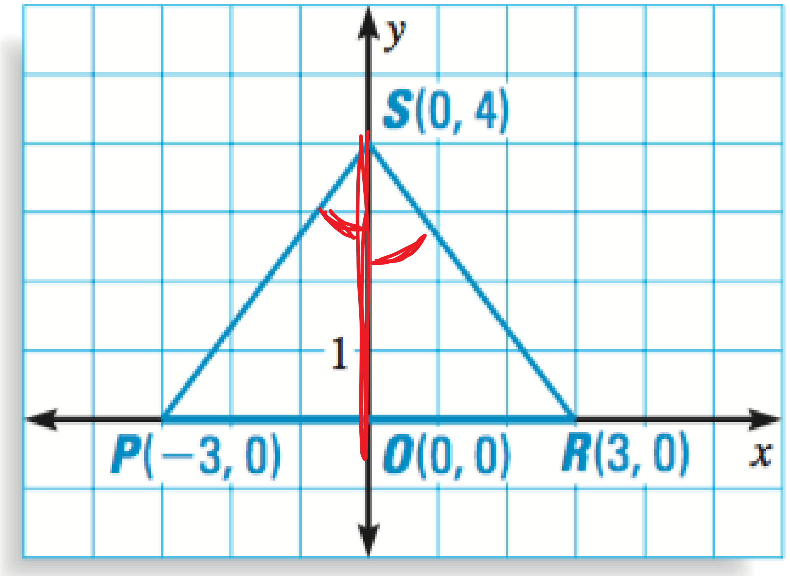
Once a figure is placed in a coordinate plane, you may be able to prove statements about the figure.

Example 4: Writing a Plan for a Coordinate Proof

Write a plan to prove that $\overrightarrow{SO}$ bisects $\angle PSR$.

GIVEN ▶ Coordinates of vertices of $\triangle POS$ and $\triangle ROS$

PROVE ▶ $\overrightarrow{SO}$ bisects $\angle PSR$

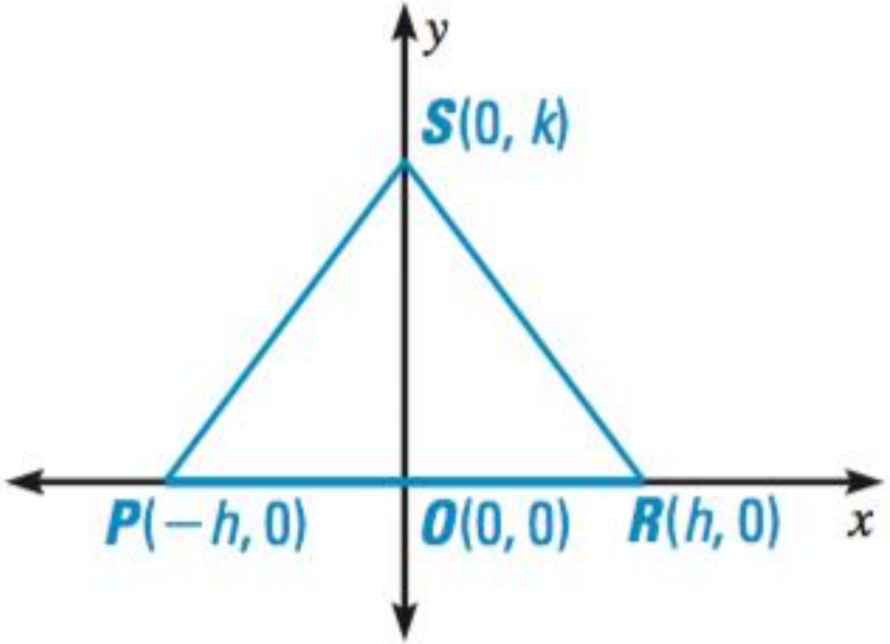


Use distance formula to show all 3 sides of Tri. SPO and Tri. SRO are congruent.

Then use CPCTC to show <PSO cong. <RSO.

The coordinate proof in Example 4 applies to a specific triangle. When you want to prove a statement about a more general set of figures, it is helpful to use variables as coordinates.

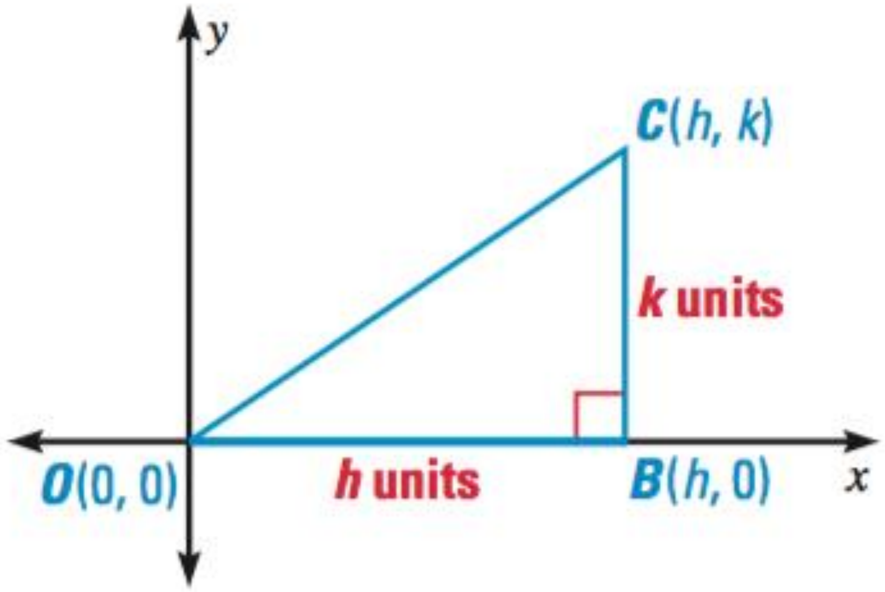
For instance, you can use variable coordinates to duplicate the proof in Example 4. Once this is done, you can conclude that $\overrightarrow{SO}$ bisects $\angle PSR$ for *any* triangle whose coordinates fit the given pattern.



Example 5: Using Variables as Coordinates

Right $\triangle OBC$ has leg lengths of h units and k units. You can find the coordinates of points B and C by considering how the triangle is placed in the coordinate plane.

Point B is h units horizontally from the origin, so its coordinates are $(h, 0)$. Point C is h units horizontally from the origin and k units vertically from the origin, so its coordinates are (h, k).



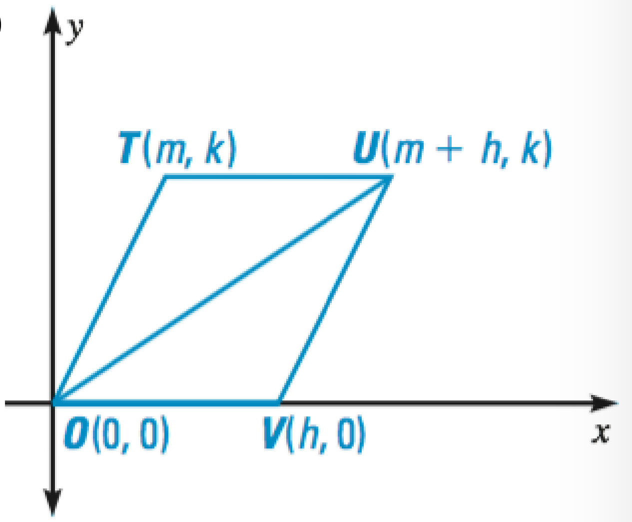
You can use the Distance Formula to find the length of the hypotenuse $\overline{OC}$.

$$OC = \sqrt{(h-0)^2 + (k-0)^2} = \sqrt{h^2 + k^2}$$

Example 6: Writing a Coordinate Pro

GIVEN ▶ Coordinates of figure *OTUV*

PROVE ▶ $\triangle OTU \cong \triangle UVO$



OU cong. UO (reflexive/O.S.)

OT cong. UV (D.F.)

$OT \rightarrow \sqrt{(m-0)^2+(k-0)^2} \rightarrow \sqrt{m^2+k^2}$

$UV \rightarrow \sqrt{(m+h-h)^2+(k-0)^2} \rightarrow \sqrt{m^2+k^2}$

TU cong. VO (D.F.)

$TU \rightarrow \sqrt{(m-m+h)^2+(k-k)^2} \rightarrow \sqrt{h^2}$

$VO \rightarrow \sqrt{(h-0)^2+(0-0)^2} \rightarrow \sqrt{h^2}$

Tri. OTU cong. Tri. UVO by SSS

EXIT SLIP